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QCD equation of state at non-zero chemical potential

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We present our new results for the QCD equation of state at nonzero chemical potential at $N_t = 6$ and compare them with $N_t = 4$. We use the Taylor expansion method with terms up to sixth order in simulations with 2+1 flavors of improved asqtad quarks along a line of constant physics with $m_l = 0.1m_s$ and approximately physical strange quark mass m_s.

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1. Introduction

Experiments at RHIC start with a baryon rich environment; hence they naturally have a non-zero chemical potential. The finite temperature field theory formalism easily admits a chemical potential; however, it leaves us with a complex action, and we can no longer use importance sampling. This results in the well known sign problem. If the chemical potential is small, we can employ the Taylor expansion method [1]. This method requires simulations only at zero chemical potential. Here we employ this method to study QCD with three dynamical quarks using the asqtad action [2] that we have already extensively studied at non-zero temperature, but without a chemical potential. Previously, we have performed a study with nonzero chemical potential only for $N_t = 4$ [3].

2. Methodology

We briefly review the formalism and methods that are detailed in Refs. [1] and [3]. Physical quantities of interest are Taylor expanded in the chemical potentials (in physical units) $\bar{\mu}_l$ and $\bar{\mu}_h$ for light and strange quarks, respectively. We drop the bar when referring to the chemical potential in lattice units. For example:

$$\frac{p}{T^4} = \frac{\ln Z}{VT^3} = \sum_{n,m=0}^{\infty} c_{nm}(T) \left(\frac{\bar{\mu}_l}{T}\right)^n \left(\frac{\bar{\mu}_h}{T}\right)^m . \tag{2.1}$$

Only terms with $n+m$ even appear due to CP symmetry.

$$c_{nm}(T) = \frac{1}{n!}\frac{1}{m!}\frac{N_\tau^3}{N_\sigma^3}\frac{\partial^{n+m}\ln Z}{\partial(\mu_l N_\tau)^n \partial(\mu_h N_\tau)^m}\bigg|_{\mu_{l,h}=0} . \tag{2.2}$$

For the interaction measure,

$$\frac{I}{T^4} = -\frac{N_t^3}{N_s^3}\frac{d\ln Z}{d\ln a} = \sum_{n,m}^{\infty} b_{nm}(T) \left(\frac{\bar{\mu}_l}{T}\right)^n \left(\frac{\bar{\mu}_h}{T}\right)^m . \tag{2.3}$$

Temperature dependendent coefficients $c_{nm}(T)$ and $b_{nm}(T)$ are combinations of observables that can be calculated on non-zero T ensembles, but with zero chemical potential. We Taylor expand up to sixth order. To compute all the required terms, 40 fermionic observables have to be determined using stochastic estimators, as well as several gluonic observables [3]. Ensembles are generated on a line of constant physics with $m_l = 0.1 m_s$ and m_s approximately the physical strange quark mass. Our previous work used lattices with $N_t = 4$. We now use $N_t = 6$ and compare with the coarser lattices. Before we present our results, it is interesting to compare the free theory for different N_t to see how the continuum limit is approached. (See Fig. 1 [3].)

Turning to the interacting theory we show the unmixed coefficients for the pressure in Fig. 2. There is considerable structure at low T and then an approach to the Stefan-Boltzmann (SB) limit above the cross-over temperature. Also, the higher order coefficients are small, but their errors grow rapidly. Note that the errors are better controlled for $N_t = 6$ (red) than they were for $N_t = 4$ (black).

In Fig. 3, we show the mixed coefficients for the pressure. Similar figures are available for the coefficients that are relevant for the interaction measure. Due to lack of space, we will not show them here.

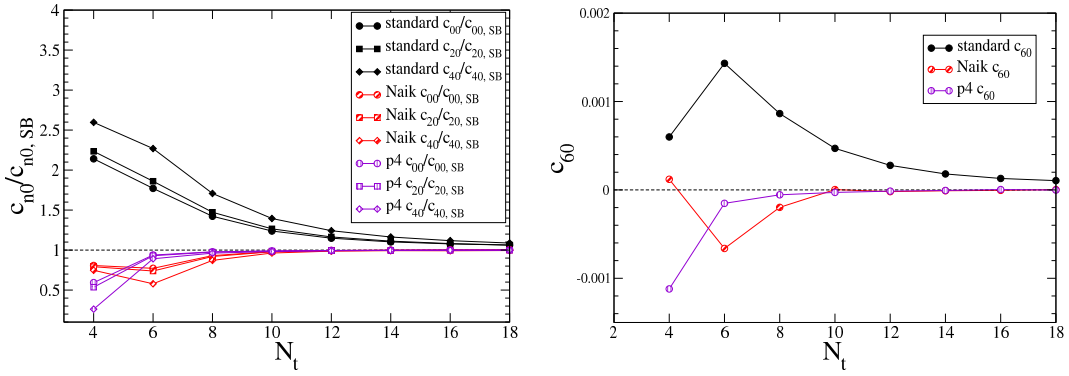

Figure 1: Several expansion coefficients as a function of N_t for the free theory [3]. On the left, for non-vanishing coefficients, we show the the ratio to the Stefan-Boltzmann limit. On the right, for c_{60}, we show the value itself.

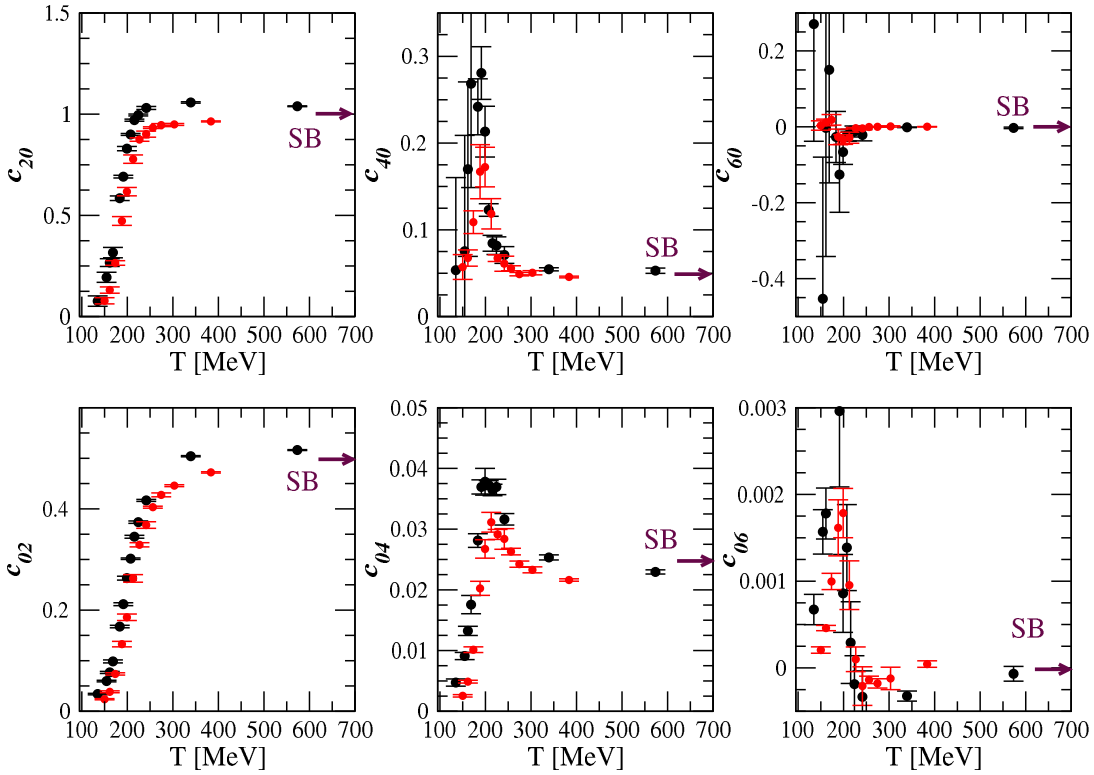

Figure 2: Unmixed Taylor coefficients c_{n0} and c_{0n} as a function of temperture. New results for $N_t = 6$ are shown in red; black is used for $N_t = 4$.

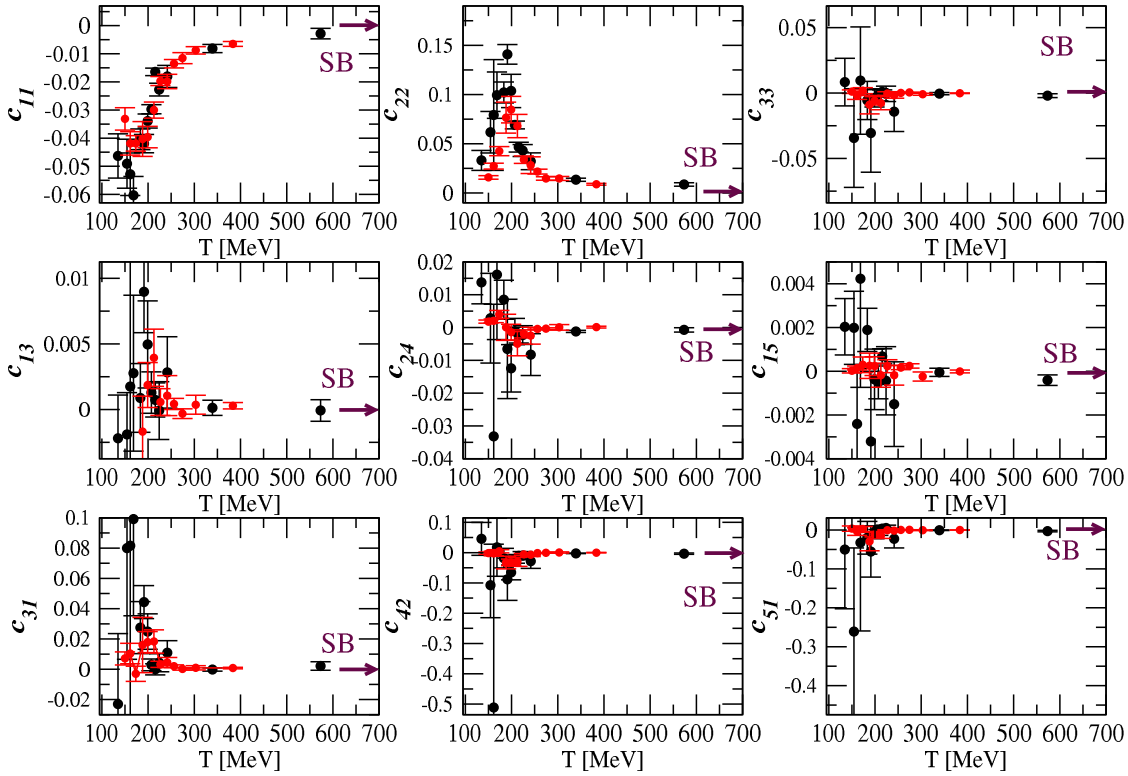

Figure 3: Mixed Taylor coefficients c_{mn} as a function of temperture. New results for $N_t = 6$ are shown in red, black is used for $N_t = 4$.

3. Results

With the coefficients in hand, we can calculate interesting quantities, such as pressure, interaction measure, energy density, quark number density, and quark number susceptibility. Due to non-zero $c_{n1}(T)$ terms a non-zero strange quark density is induced even with $\mu_h = 0$. To study the $n_s = 0$ plasma, we must, therefore, tune μ_h as a function of μ_l and T.

In Figs. 4 and 5, we show how the pressure and interaction measure change as a function of T for selected values of μ/T. In each figure, we display the zero chemical potential case on the left. We compare results for $N_t = 6$ with our prior results for $N_t = 4$. We find that the change in pressure is somewhat smaller compared with our previous results. For the interaction measure, the errors are fairly large, but there also seems to be a reduction there. Figure 6 shows the energy density and change in energy density due to chemical potential. In this case, the differences between $N_t = 4$ and 6 are small. In Fig. 7, we show the light quark number density and the quark number susceptibility. We note that both the number density and susceptibility are somewhat smaller with $N_t = 6$ than they were for $N_t = 4$.

One particularly interesting quantity is the isentropic equation of state (EOS). In a heavy-ion collision, after thermalization, the system expands and cools with constant entropy. Thus, we would like to find the EOS with fixed ratio of entropy to baryon number. The appropriate ratio of s/n_B for AGS, SPS and RHIC are 30, 45 and 300, respectively. To carry out this calculation we must

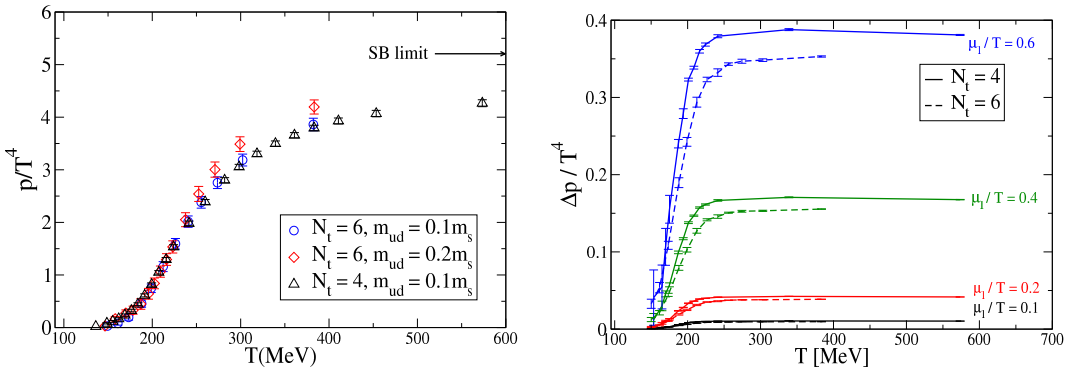

Figure 4: The pressure with zero chemical potential (left) and the change in pressure due to chemical potential (right).

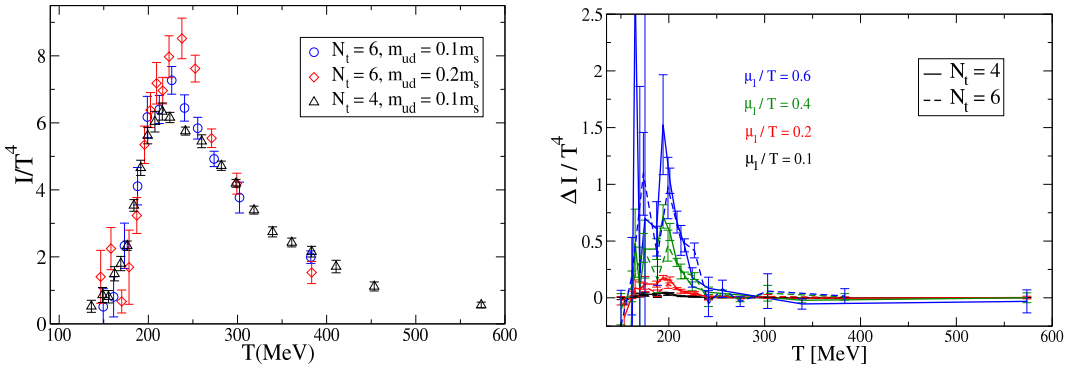

Figure 5: The interaction measure with zero chemical potential (left) and the change in interaction measure due to chemical potential (right).

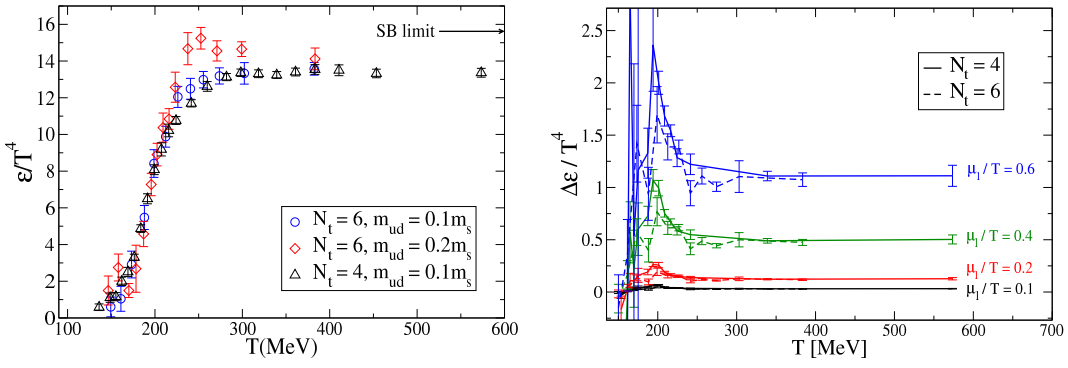

Figure 6: The energy density with zero chemical potential (left) and the change in energy density due to chemical potential (right).

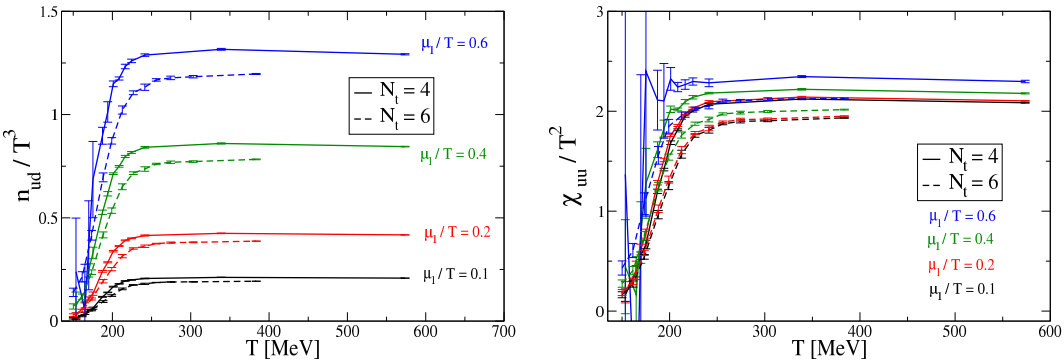

Figure 7: Light quark number density (left) and quark number susceptibility (right).

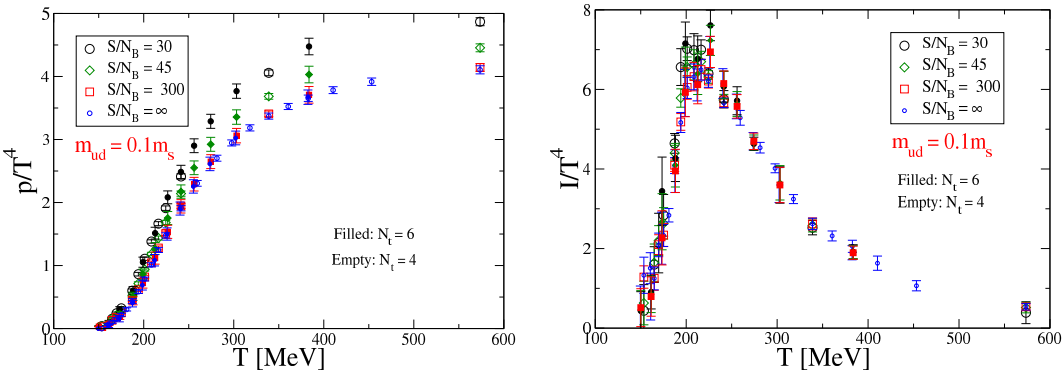

Figure 8: Isentropic pressure (left) and interaction measure (right) for selected values of S/n_B appropriate to AGS, SPS and RHIC.

find trajectories in the (μ_l,μ_h,T) space with $n_s = 0$ and s/n_B as stated above. In Fig. 8, we show the isentropic pressure and interaction measure. We also show the isentropic energy density, and quark number density in Fig. 9, and the isentropic light and strange quark susceptibilities in Fig. 10.

4. Conclusions

We have extended our sixth order Taylor expansion study of thermodynamics with chemical potential toward the continuum limit by going from $N_t = 4$ to 6. After computing the expansion coefficients relevant for both pressure p and interaction measure I we can compute a number of interesting quantities. We observe modest lattice spacing effects, with the quark densities and susceptibilities, and the effect of chemical potential, smaller at the smaller lattice spacing. In addition, we have calculated the isentropic equation of state, which is particularly relevant for the phenomenology of relativistic heavy-ion colliders. For both values of N_t we find rather smooth behavior for the isentropic variables indicating that experiments are far from any critical point in the μ–T plane.

It would be interesting to extend this work to yet smaller lattice spacing and to go to lighter quark mass.

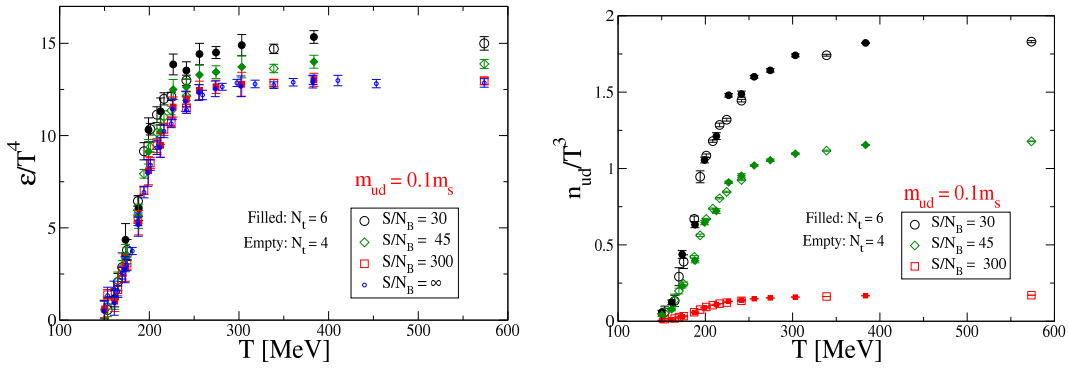

Figure 9: Isentropic energy density (left) and light quark number density (right) for selected values of S/n_B appropriate to AGS, SPS and RHIC.

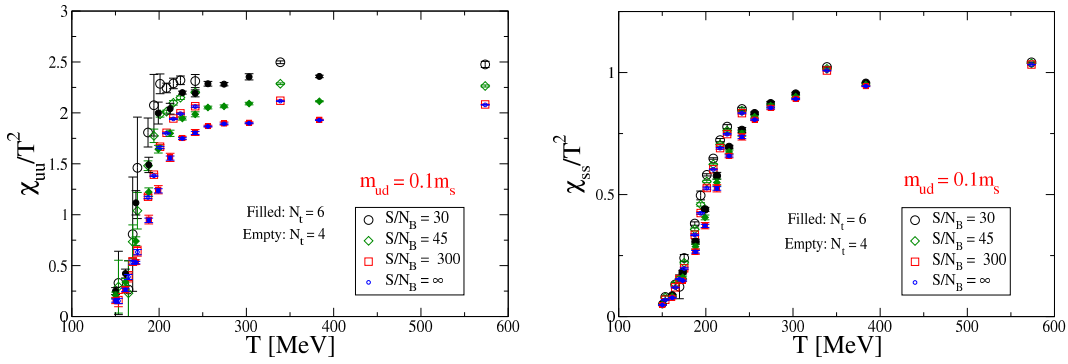

Figure 10: Isentropic light quark number susceptibility (left) and strange quark number susceptibility (right) for selected values of S/n_B appropriate to AGS, SPS and RHIC.

Acknowledgements

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